Timelike Hypersurfaces in Causal Sets

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Overview

We must understand the action in order to use the path integral approach, learn the phase structure, and identify fixed points & critical exponents

- Brief background on causal sets
- Causal set action: what do we know?
- Problems with timelike boundaries
- Measuring boundary geometry: theory
- Measuring boundary geometry: numerics
- Current work and open problems

1. Causal Sets
2. Divergence of the Action
3. Measuring Curvature: Theory
4. Measuring Curvature: Numerics
5. Open Problems

Causal Sets

Discrete spacetime is represented by a directed acyclic graph (DAG): nodes are fundamental elements of spacetime and relations indicate causality.

- Hawking/Malament: causal structure is enough to recover topology
- Many results on kinematics - all geometric structure should be encoded in the graph
- All causal sets fall into two categories: manifoldlike (random) and non-manifoldlike (crystalline)
- Discretize a continuum space via Poisson point process (sprinkling) into a compact region
- General causal sets generated using a Markov chain
- These phases are separeated by a first order phase transition in 2D
- The discreteness scale is fundamental, not a regularizer

Causal Set Action

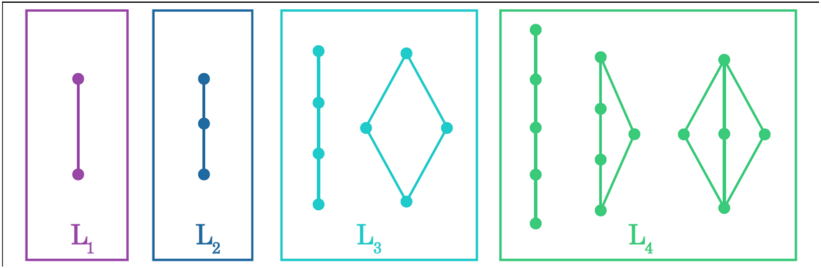
The causal set action was developed over a number of years, first for the bulk, then for spacelike boundaries.

- The action depends on the spacetime dimension
- One measures ($O(N^3)$) the abundance of primitive subgraphs
- Boundary terms are not included in the "bulk" expression
- A separate expression exists for dealing with spacelike boundaries

Unknown: How to deal with timelike boundaries and co-dimension 2 joints? Essential to fully understand this in order to do Monte Carlo experiments.

Causal Set Action: Measurement

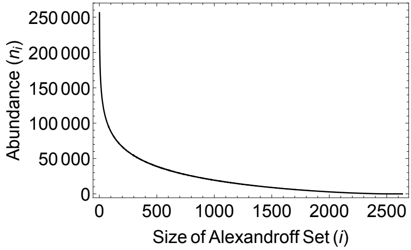
To measure the action, we count the number of these primitive subgraphs, called *order intervals*:

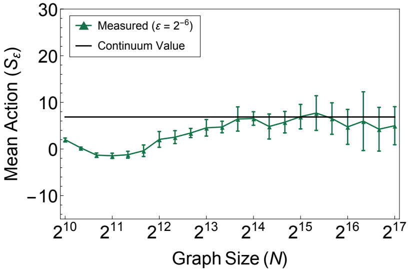


One may reduce fluctuations for finite systems by "smearing" over a mesoscale, effectively using all primitive subgraphs.

Interval Abundance Distribution

$$S_2(C) = 2(N - 2n_1 + 4n_2 - 2n_3)$$

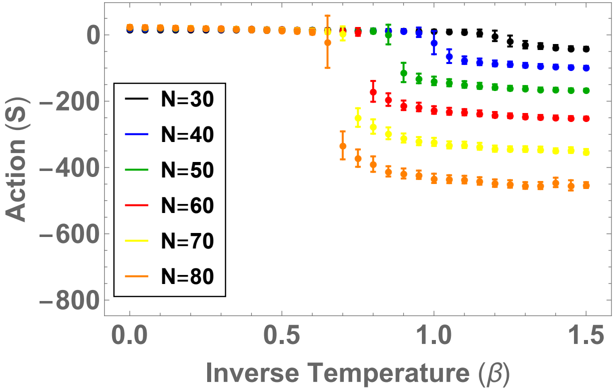




- Curve is characteristic, perhaps unique, for a spacetime
- Convergence is slower for higher dimension, curved spacetimes

Partition Function and 2D Phase Transition

$$Z(N, d, \mathcal{T}) = \sum_{C} e^{-\beta S_d(C)/\hbar}$$



Current experiments do not include any boundary terms.
How important are they?

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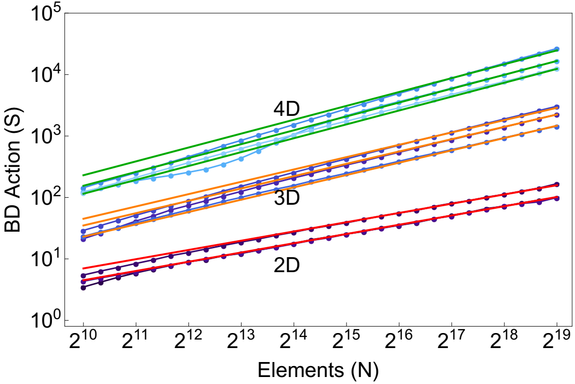
Timelike Boundaries

The original derivation of the action explicitly assumes convexity of the region: only spacelike or null boundaries allowed

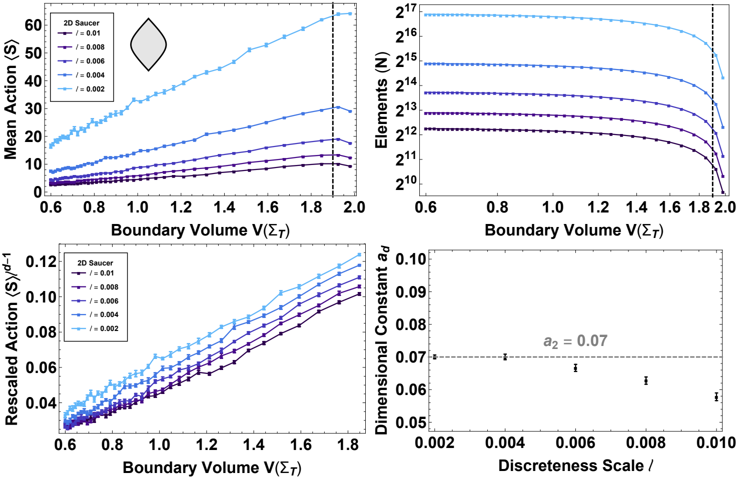


Regions: (2D) Square, Saucer; (3D/4D) Cube, Cylinder, Half Diamond

Observed Fixed-Volume Scaling: $S \sim N^{(d-1)/d} \sim (1/\ell)^{d-1}$

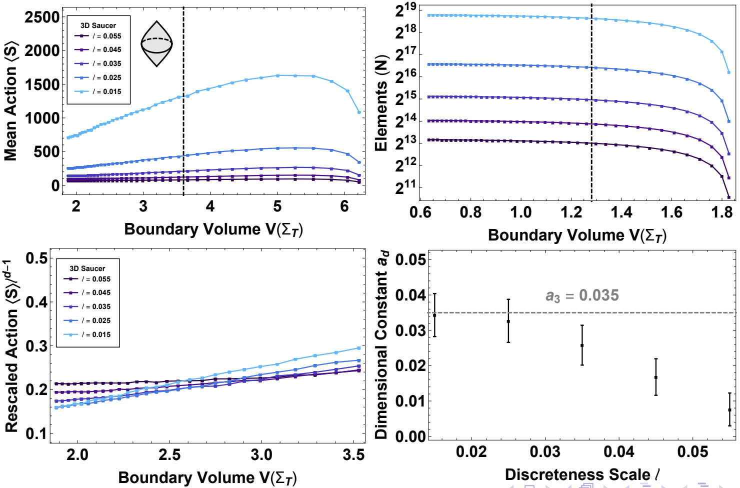
Characterizing the Divergence: 2D

$$\text{Ansatz: } S_{div} = a_d V(\Sigma)/\ell^{d-1}$$



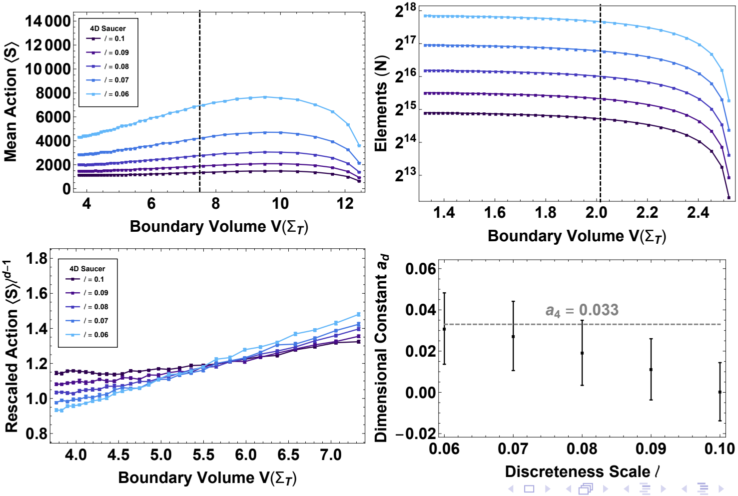
Characterizing the Divergence: 3D

Ansatz: $S_{div} = a_d V(\Sigma)/\ell^{d-1}$



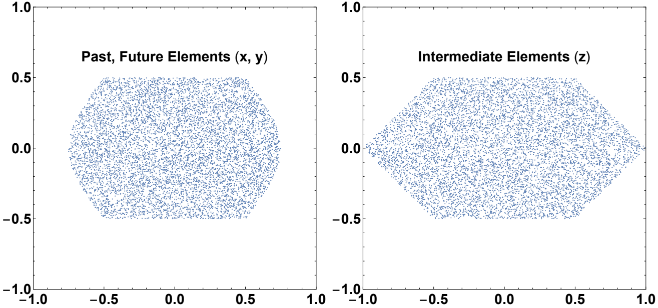
Characterizing the Divergence: 4D

Ansatz: $S_{div} = a_d V(\Sigma)/\ell^{d-1}$



Partial Solution: Nullification

The divergence disappears when we consider x, y in the region of interest and z (such that $y \prec z \prec x$) in the nullified region:



- We only need to know the *number* of missing elements in each order interval, but *not their internal ordering*
- The number of missing elements in intervals near the boundary tells us something about the extrinsic curvature of

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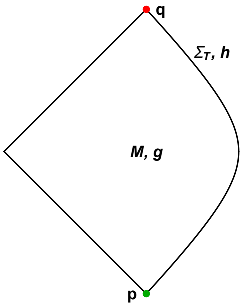
Intervals Near the Boundary (Continuum Geometry)

For a flat boundary in flat 2d space, $V(p, q) = T^2/4$. Expand V in terms of T to order T^{d+1} for a general spacetime with a general boundary.



Geometric observables are evaluated at the midpoint on the boundary.

The First Order Correction

Dimensional Analysis: $V(T) = V_{\text{flat}}(T)[1 + \mathcal{G}T + O(T^2)]$

- It can be shown $\mathcal{G} = c_1\mathcal{S}_1 + \ldots + c_n\mathcal{S}_n$; $\mathcal{S}_i$ are all independent scalars involving a single derivative of local geometric quantities: metric $g_{\mu\nu}$, tangent vector v^μ, normal vector n^μ
- Trivially, $\nabla_\mu g_{\nu\lambda} = 0$
- $n^\mu \nabla_\mu n_\nu$ is not well defined
- It can be shown $v^\mu n^\nu \nabla_\mu n_\nu = 0$, $v^\mu v^\nu \nabla_\mu v_\nu = 0$
- Extrinsic curvature: $K = \nabla_\mu n^\mu$
- Curvature tensor: $v^\mu v^\nu \nabla_\mu n_\nu = K_{ab} v^a v^b$, $v^\mu n^\nu \nabla_\mu v_\nu = -K_{ab} v^a v^b$

$$V(T) = V_{\text{flat}}(T)[1 + (c_1 K + c_2 K_{ab} v^a v^b)T + O(T^2)]$$

Correction Coefficients

We now solve for the coefficients c_1 and c_2

- In 2D, K and K_{ab} are not independent
- $V(T) = V_{\text{flat}}(T)[1 + c_1 KT + O(T^2)]$
- Taking a constant-curvature surface, we find $V(T) = \frac{T^2}{4}\left[1 - \frac{1}{3}KT + O(T^2)\right]$
- In 3D, we must consider multiple spacetimes to solve for c_1, c_2
- With some algebra, we find $V(T) = \frac{\pi T^3}{24}\left[1 + \frac{1}{\pi}\left(K_{ab}v^a v^b - \frac{1}{4}K\right)T + O(T^2)\right]$
- 4D result in progress

Necessary assumptions:

- T is small relative to bulk curvature, $RT^2 \ll 1$
- T is small relative to extrinsic curvature, $KT \ll 1$

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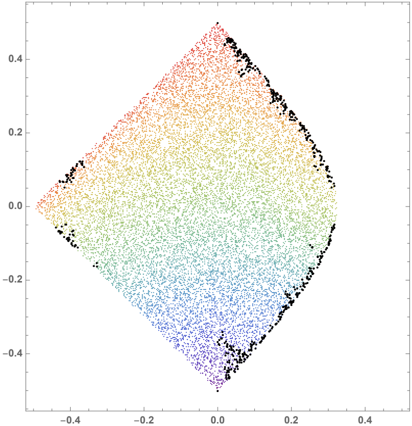
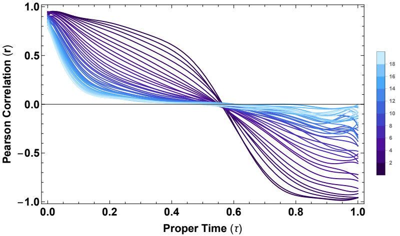
Identifying Timelike Boundaries

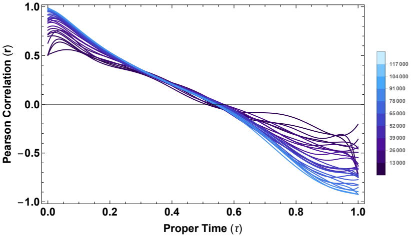
Partition a region with a constant-curvature timelike boundary into spatial layers:



Identifying Timelike Boundaries

The number of relations is correlated with the spatial position:

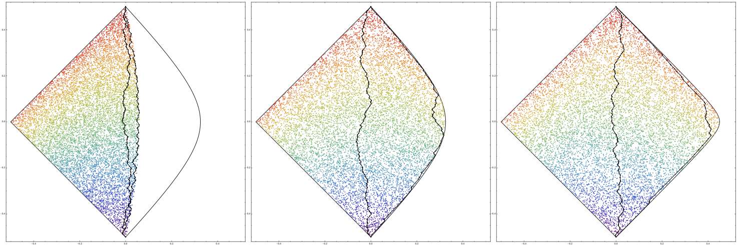




Using this, we select elements with few relations when the correlation is negative.

Identifying Timelike Boundaries

We measure the *longest chain* and the *longest boundary chain* using this algorithm

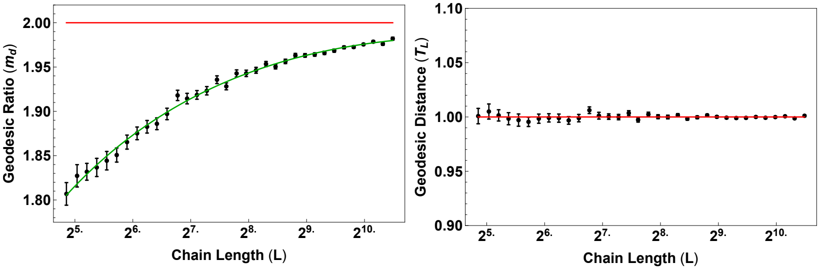


From these, we wish to measure the proper time T_M as well as the boundary length T_Σ as $\ell \to 0$.

Previous: $T \propto L\ell$ (Brightwell & Gregory 1992)

Convergence of Longest Chains

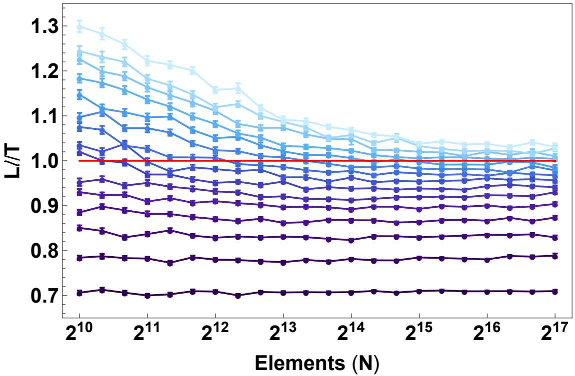
In a 2D Minkowski Diamond...



Hence we may accurately measure T using finite-sized chains using the fit. Convergence is $\log\log L$

Convergence of Boundary Chains

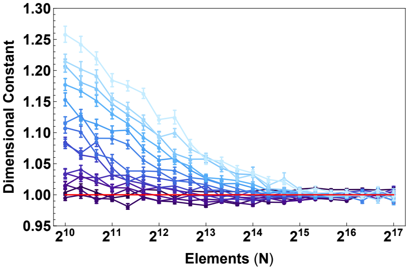
For a 1D boundary, we expect $T = L\ell$

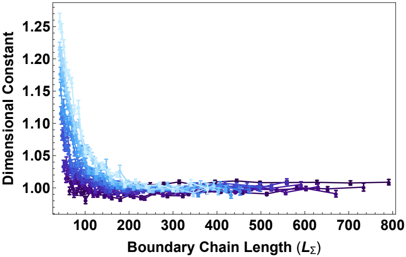


Problem: We find it converges to something dependent on K

Convergence of Boundary Chains

We instead find $T = L\ell/(a \log K + b)$





We find similar results for maximal chains through the center!

We need to better understand convergence of non-geodesic paths using one side of a partition.

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Current/Future Work

Divergence:

1. Analytic expression for the divergence in a 2D square
2. High-precision data on the proportionality coefficient a_d
3. Rates of convergence (divergence?)

Curvature:

1. Methods to measure curvature tensor $K_{\mu\nu}$ and tangent vectors in 3D
2. Design experiments in 3D where $K_{\mu\nu} = 0$ but $K \neq 0$.
3. Approximation for the half-cone volume in 4D

Timelike (Non-Geodesic) Curves:

1. How do maximal chains converge in the half diamond?
2. What procedure can we use to study the convergence of discrete paths to a specified continuum curve?
3. Can we prove similar results in Riemannian (Euclidean) space?

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