Introduction to Network Science

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Network Science Institute
Northeastern University

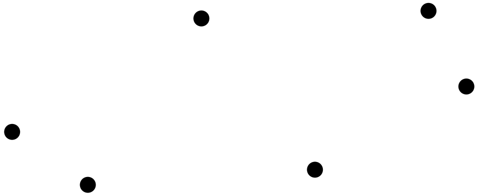
November 30, 2017

1. Introduction to Networks
   - Basic Concepts
   - Models of Networks
   - Real Network Properties

2. Network Dynamics
   - Diffusion
   - Epidemics
   - Navigation

What is a Network?

A network is a set of **nodes**



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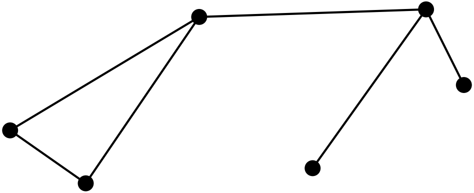
A network is a set of **nodes** and **links**:



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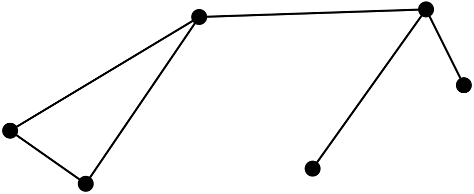
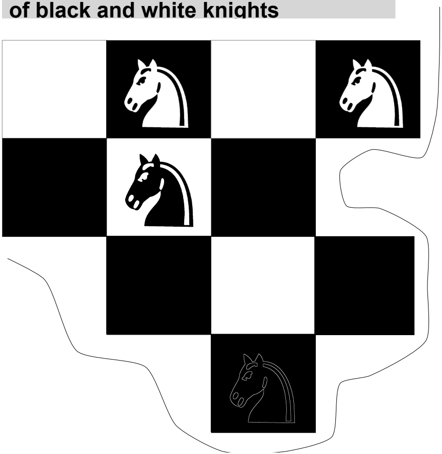


In mathematics, we call this a **graph**.

A Quick Problem

A chess puzzle: swap the positions of black and white knights



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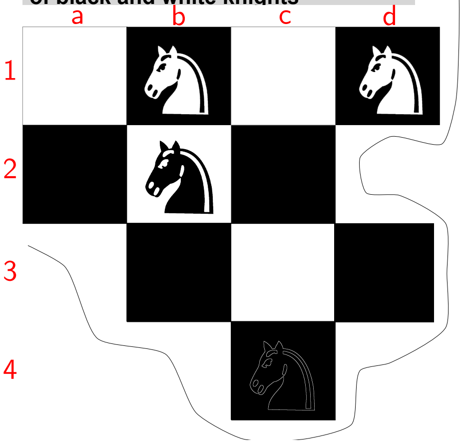

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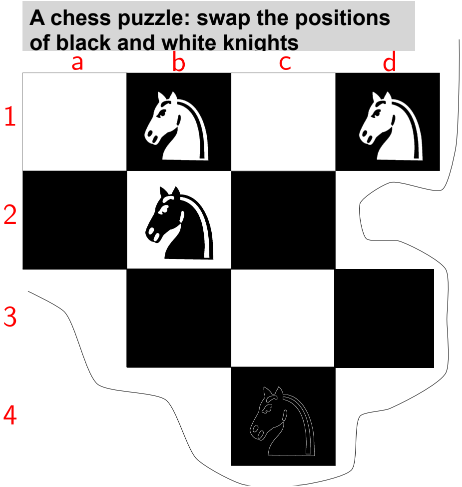

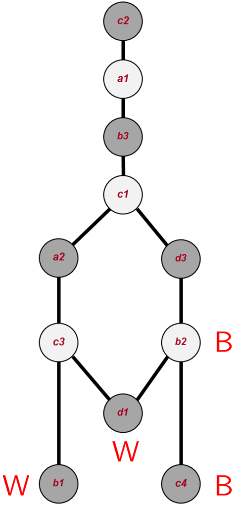

What is a Network?

Who cares?

Network Science

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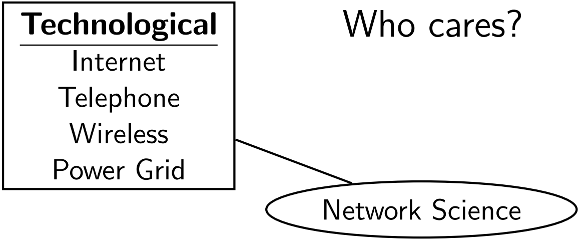

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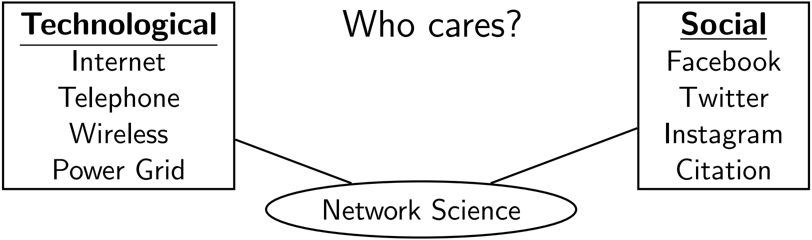

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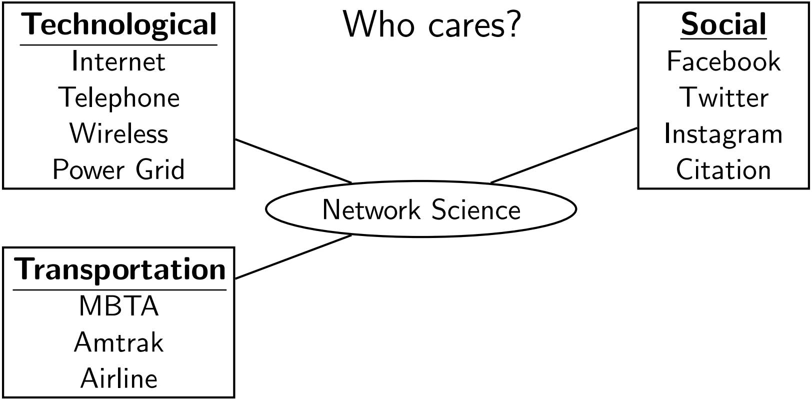

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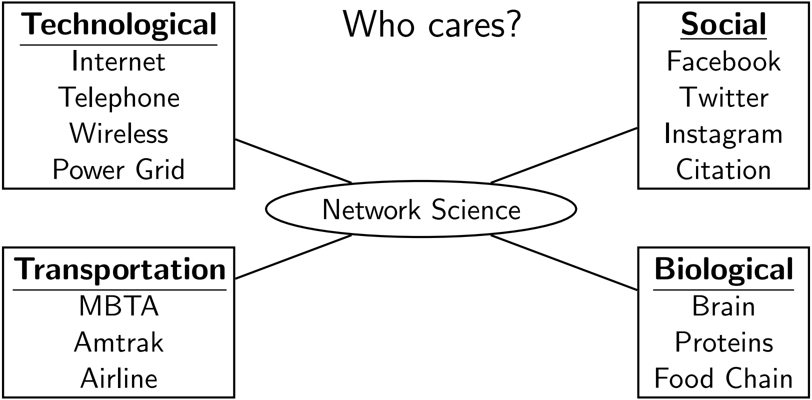

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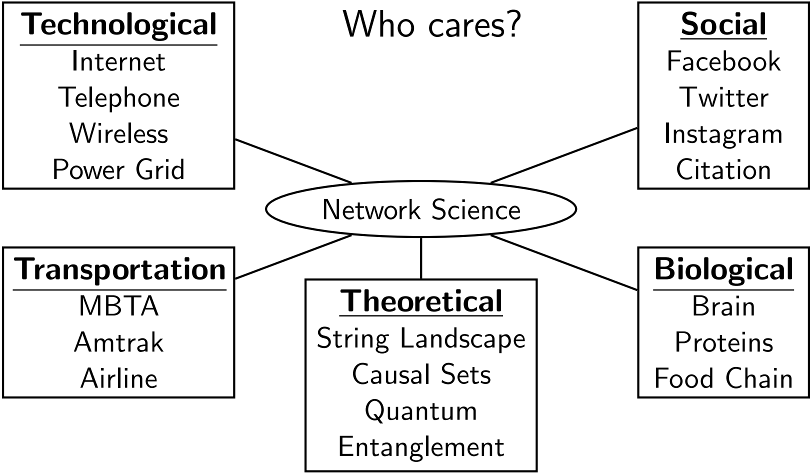

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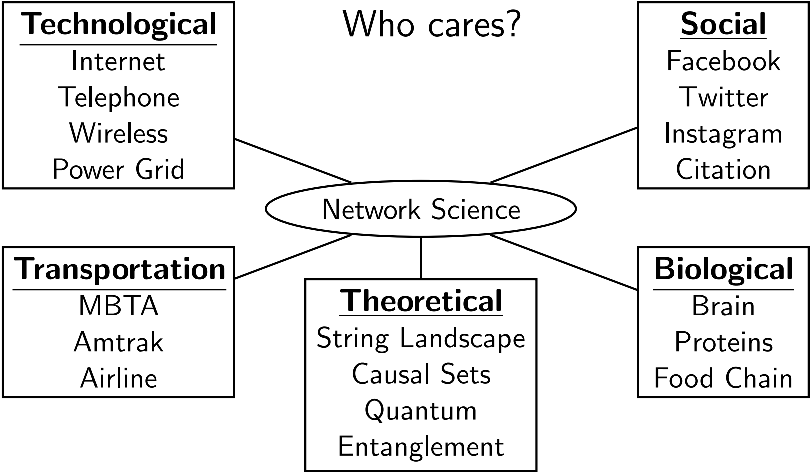
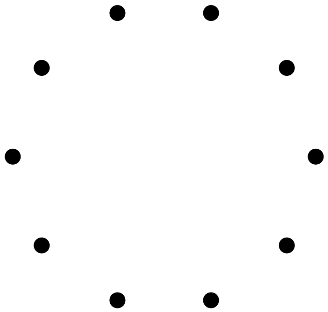


Goal: control, predict, and understand.

The Erdős-Rényi Random Graph

To talk about probability and statistics, we need a null model:
$G(N, p)$ or $G(N, M)$



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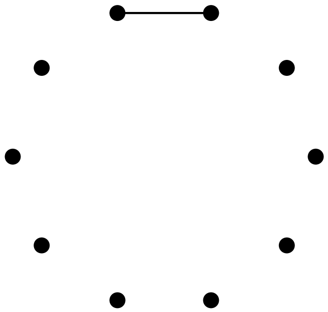

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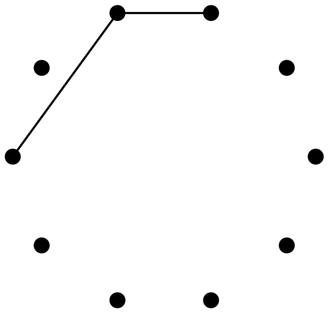

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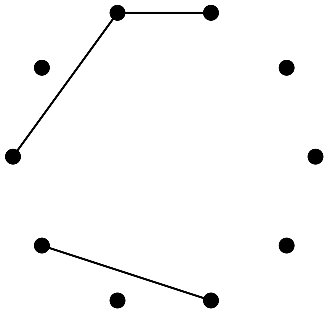

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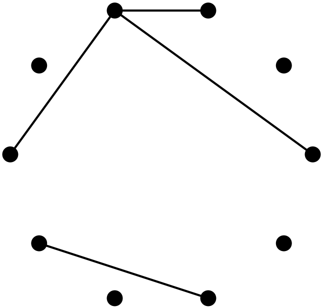

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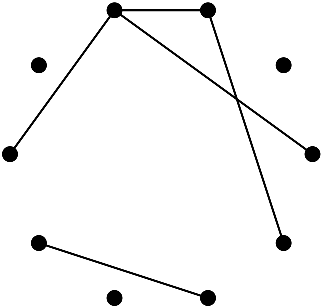

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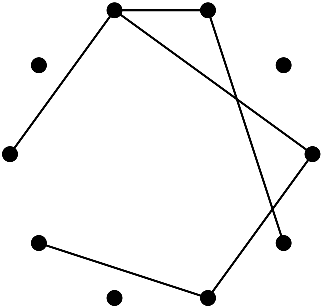

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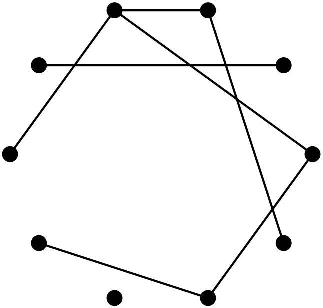

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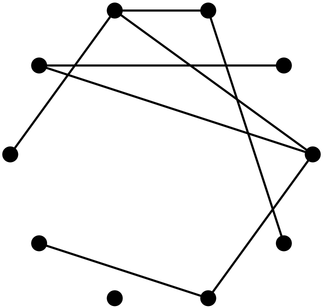

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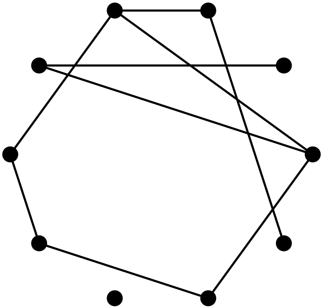

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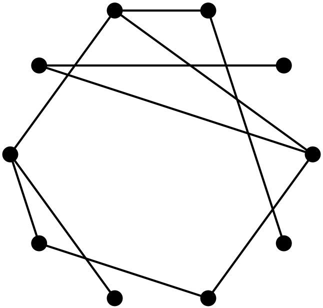

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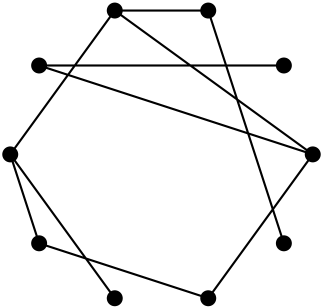
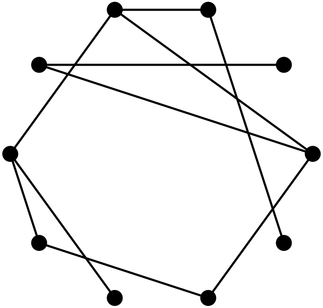


Nodes in real networks do not connect randomly!

Basic Structural Properties

How do we characterize a network?

Degrees and Clustering



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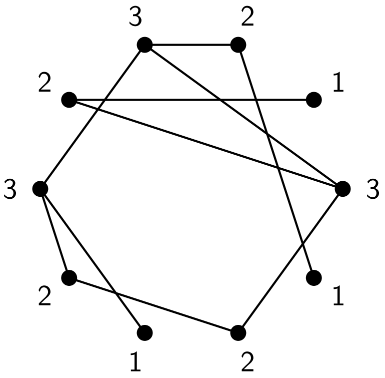
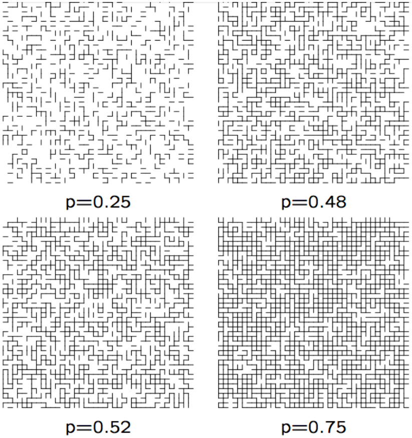


Percolation in Graphs



The Barabási-Albert Model

Real networks are modeled by “preferential attachment”:

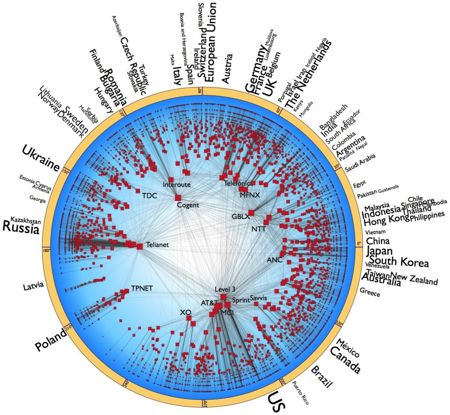


Image: *Boguñá, Papadopoulos & Krioukov, Nat. Comm. 1, 62 (2010).*

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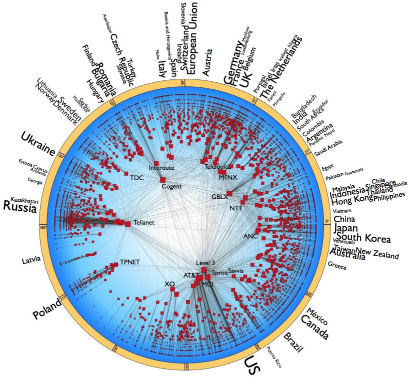

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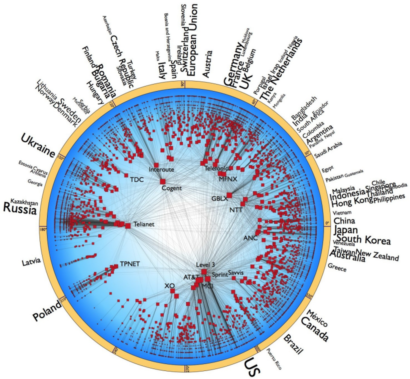

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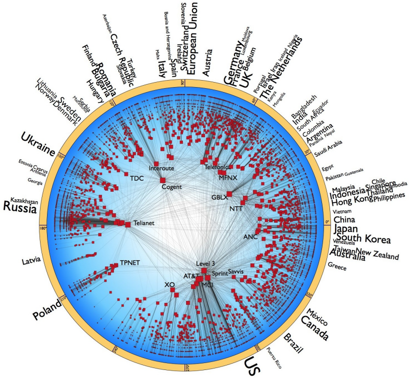

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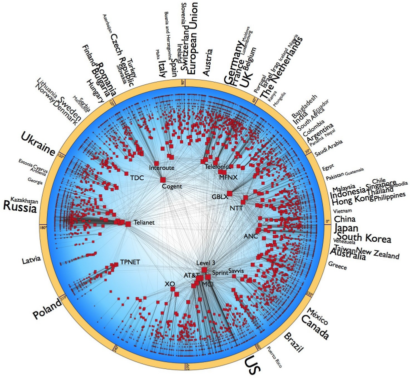

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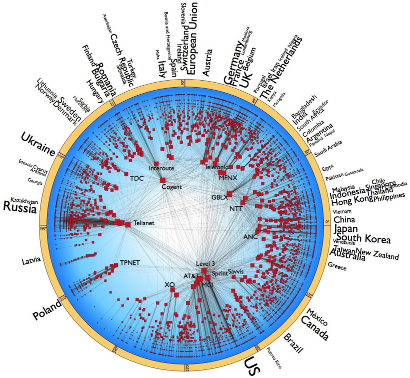

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4. At late times, $\gamma = 3$
5. **The rich get richer**

This isn't the whole story...

Degree Distributions

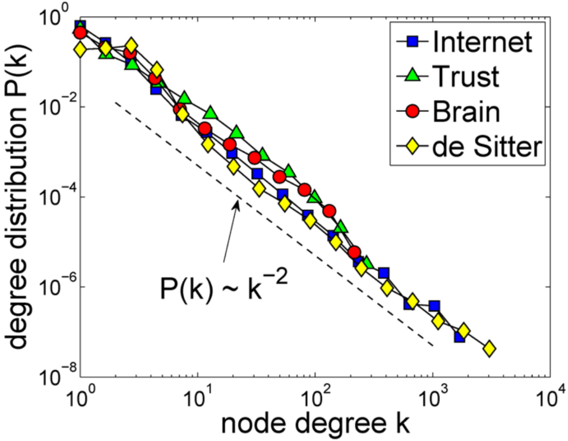
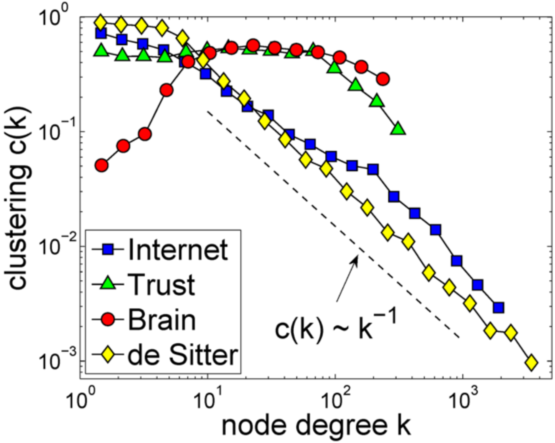


Image: *Krioukov et al., Sci. Rep. 2, 793 (2012).*

Clustering



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Processes on Networks

1. Diffusion
2. Interactions
3. Navigation
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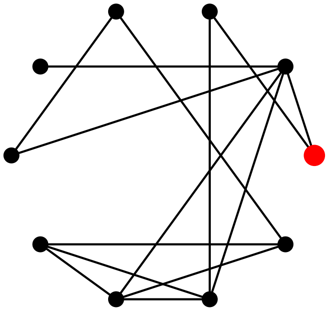
Often we see a combination of both types.

Diffusion

Diffusion is modeled as a random walk.



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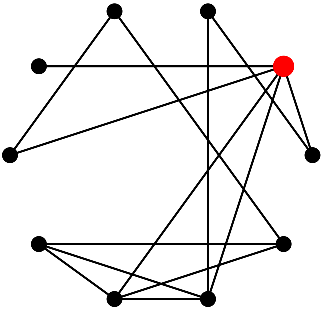

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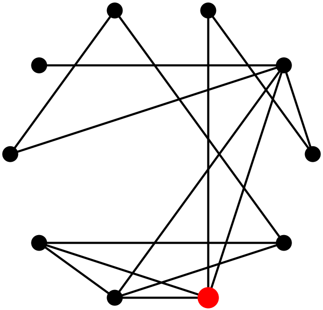

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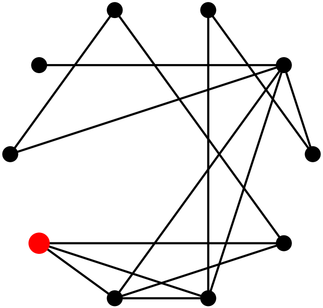

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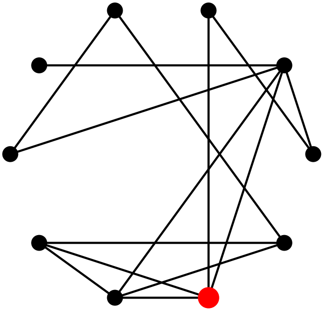

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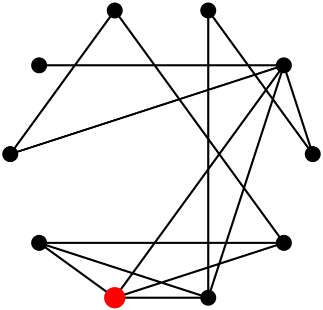

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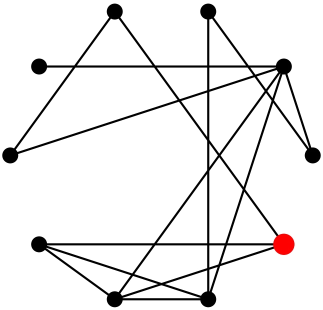

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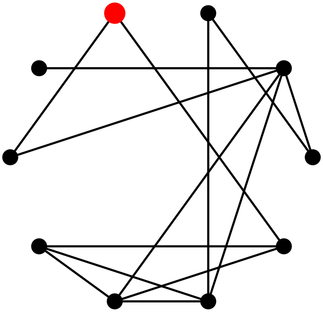

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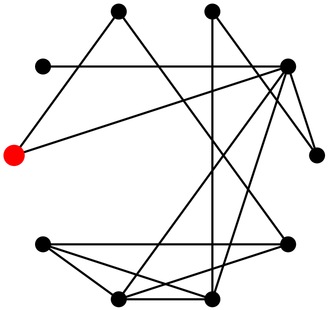

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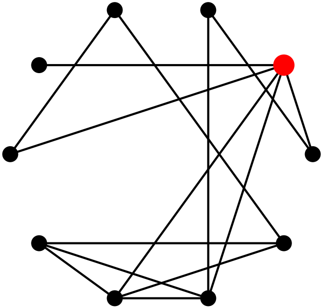

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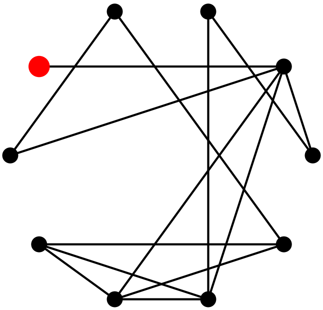

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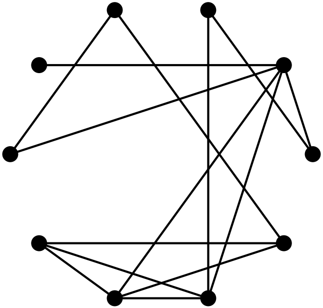


In reality, we have many objects walking, $W \gg N$.

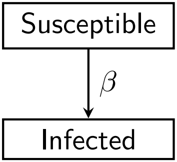
Epidemics: SIR Model

Epidemics: diffusion and mixing over time

Susceptible

Epidemics: SIR Model

Epidemics: diffusion and mixing over time



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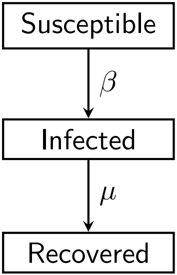

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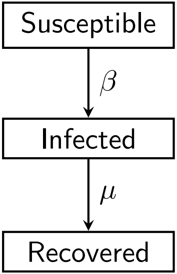

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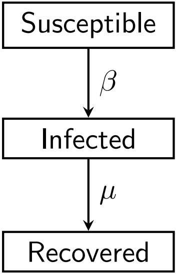

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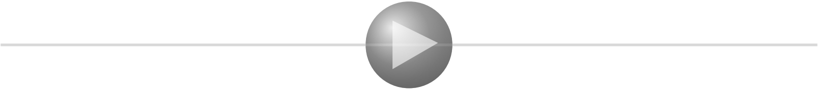
$$\frac{dr(t)}{dt} = \mu i(t)$$

Epidemic Timescale: $\tau = \frac{\langle k \rangle}{\beta \langle k^2 \rangle - (\mu + \beta) \langle k \rangle}$

H1N1 Epidemic (2009)



Credit: *A. Vespignani*

Navigation

Navigation uses latent geometry:

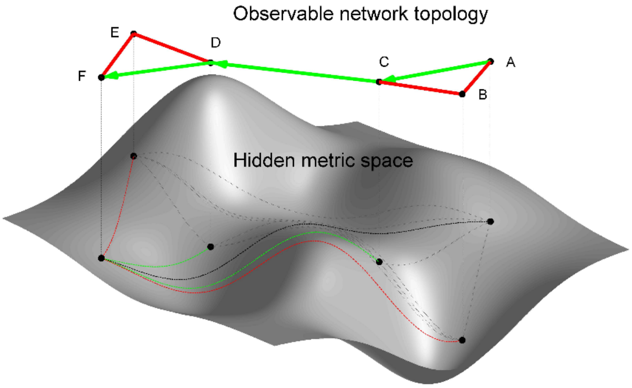
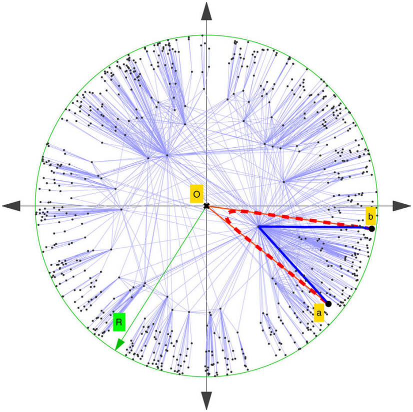


Image: *D. Krioukov*

Navigation

Navigation uses latent **hyperbolic** geometry:



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Summary

- Networks are powerful models of complex systems
- The same techniques are applicable to seemingly unrelated fields
- Computational tools are available to easily simulate/analyze networks
- Scalability is important for larger problems